On numerical and Mathematical description of premixed flame dynamics

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Combustion is the oldest technology for energy generations in human history. Although renewable energy is getting one of the mainstream of energy generations, combustion research still occupies an important position for balancing energy resources and improving energy efficiency. Studies involving flame dynamics (an essence for understanding combustion process) over the past decades have been conducted using a variety of numerical and perturbative (mathematical) methodologies. Essential insights on flame structure resulted from Darrieus [1] and Landau [2], the early work of Markstein [3], Matalon and Matkowsky [4] and some other works based on asymptotic analysis and various numerical simulations. Other early studies of flame dynamics were predominantly based on weakly-nonlinear models of the Michelson-Sivashinsky type [5] that assume that the densities of the burnt and unburnt gases vary only slightly from each other. Such simplified models have provided *mathematical insights* in flame dynamics, which helped developing the more comprehensive studies based on the hydrodynamic model that fully accounts for thermal expansion.

In the present study, we discuss the effect of gravity on dynamics of perturbed planar flames to understand flame morphology in more realistic situation. The study is threefolds:

1. **linear stability analysis** (based on *perturbation theory*),
2. **fully nonlinear analysis** (based on *numerical simulations of the hydrodynamic model*), and
3. **weak nonlinear analysis** (based on *asymptotic analysis, bifurcation theory and rigorous numerics*).

One of key issues to understand flame dynamics involving energy generation in combustion process is the *flame propagation speed* U for asymptotically stationary flame fronts depending on thermal expansion σ, a physico-chemical parameter called *Markstein number* $\mathcal{M}$ and gravity parameter G. We describe the relationship among them on *bifurcation diagrams*, which visualize information of solution structures in given systems depending on parameters. Through numerical and mathematical arguments based on bifurcation analysis, we obtain physical and mathematical understandings of flame morphology not only in realistic situation but also in severe environments with, say extremely high pressure or microgravity (space).

The present talk is based on the joint work with Prof. Moshe Matalon and Mr. Shikhar Mohan (University of Illinois at Urbana-Champaign) [6, 7, 8].

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